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Equilibrium Scour Depth at Tidal Inlets

by Steven A. Hughes

PURPOSE: The Coastal Engineering Technical Note (CETN) herein introduces a simple expression relating maximum discharge per unit width at a location in a tidal inlet to the depth of scour at that location. Application of this provisional guidance is illustrated by three examples.

BACKGROUND: One scour problem of concern at improved navigation inlets occurs where the maximum depths of the equilibrium inlet throat cross section are adjacent to a stabilizing jetty structure. These deep portions along a jetty have the potential to undercut the structure toe and cause subsequent damage to the structure armor layer.

Within the inlet channel, tidal currents play a major role in erosion and deposition of sediment. If scour occurs close to the seaward end of the inlet, wave action and longshore currents also contribute to the scour action and perhaps dominate the process. However, where scour occurs well inside the entrance channel, wave action is reduced; and it is reasonable to assume sediment movement at that location is driven primarily by the tidal flow.

Over many tidal cycles, scour in regions with minimum wave action will eventually reach a “live-bed” equilibrium depth where the maximum shear stress acting on the bottom is no longer sufficient to initiate scour of the bed. Additional scour can occur only if the maximum flow discharge is increased at that particular location. Flow increases might occur because of an overall increase in tidal prism or because of flow redirection resulting from structure alterations, dredging activities, or channel realignment.

In this technical note, a new relationship for use at tidal inlets is developed for the maximum tidal flow discharge per unit width as a function of the water depth and sediment characteristics. Measurements of maximum discharge from Ponce de Leon Inlet and Shinnecock Inlet are used to establish an upper-bound empirical coefficient. This equilibrium discharge relationship implies that there is an equilibrium depth that can tolerate a given discharge per unit width. Increases in discharge will result in scour and a corresponding increase in water depth. Practical applications of this simplified engineering approximation are suggested.

FORMULATION: Assume the vertical velocity profile at times near the maximum discharge through a tidal inlet can be represented as a steady, fully developed, rough, turbulent boundary layer extending from the bottom to the free surface. Any contribution by waves is neglected. The boundary layer velocity profile can be adequately approximated by a 1/8 power curve (Yalin 1971) with the shear stress at the bed given as

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14. ABSTRACT
This Coastal Engineering Technical Note (CETN) introduces a simple expression relating maximum discharge per unit width at a location in a tidal inlet to the depth of scour at that location. Application of this provisional guidance is illustrated by three examples

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$$t_o = r_w \left[\frac{\overline{V}}{C_k \left(h / d_e \right)^{1/8}} \right]^2 \tag{1}$$

where

$\tilde{n}_w$ = mass density of water

$\overline{V}$ = depth-averaged velocity

C_k = undetermined constant

h = water depth at maximum discharge

d_e = median grain-size diameter

The constant C_k is a boundary layer shape factor that includes the unknown relationship between d_e and bottom roughness.

The *Critical Shear Stress* of the noncohesive sand bed is given by the Shields parameter as

$$t_{cr} = C_s \left(r_s - r_w \right) g\, d_e \tag{2}$$

where

C_s = constant of proportionality

$\tilde{n}_s$ = mass density of sand

g = gravitational acceleration

d_e = median grain-size diameter

For live-bed equilibrium, a shear stress balance is assumed with $\hat{o}_o$ proportional to $\hat{o}_{cr}$. Equating Equations 1 and 2 results in the expression

$$\frac{h}{d_e} = \frac{1}{\left(C_e \right)^8} \left[\left(\frac{r_w}{r_s - r_w} \right) \left(\frac{\overline{V}^2}{g\, d_e} \right) \right]^4 \tag{3}$$

where the two unknown constants, C_k and C_s, have been combined into C_e. The term in square brackets on the right-hand side of Equation 3 is the ratio of grain-size Froude number to the immersed specific gravity of the sand, and it is defined as the *Grain Mobility Number* (Yalin 1971).

A more useful form of Equation 3 is obtained by multiplying both sides by h^8 and rearranging to get an expression for the equilibrium discharge per unit depth, i.e.,

$$q_e = C_e \left[g\left(S_s - 1\right)\right]^{1/2} d_e^{3/8} h^{9/8} \tag{4}$$

where the q_e is defined as the **Equilibrium Maximum Discharge per unit width**, given by

$$q_e = \overline{V} h \tag{5}$$

and $S_s = \tilde{n}_s/\tilde{n}_w$ is the sediment specific gravity (about 2.65 for quartz sand). As expected, Equation 4 indicates that the equilibrium maximum discharge is primarily a function of water depth with sediment size having a relatively minor effect.

MEASUREMENTS: The unknown coefficient in Equation 4 was empirically evaluated by comparision to field measurements at two dual-jetty tidal inlets. Vertical profiles of horizontal velocity were measured along transects at Shinnecock Inlet, New York, and at Ponce de Leon Inlet, Florida, using a boat-mounted acoustic Doppler current profiler. Discharge per unit width was estimated from the measurements by integrating the velocity profiles over the depth. Profiling transects across the inlet throats occurred at or around the maximum ebb or flood flow.

The results are shown in Figure 1 where calculated discharge per unit width is plotted versus the term $\left(\left[g\left(S_s - 1\right)\right]^{1/2} d_e^{3/8} h^{9/8}\right)$ on the right-hand side of Equation 4. Grain size for the Shinnecock Inlet channel was taken as 0.6 mm, whereas a size of 0.21 mm was used for Ponce de Leon Inlet. Both sands were assumed to have the same density as quartz.

The data points on Figure 1 show a wide range of discharge per unit width measured at the different depths. However, there is an upper limit to the data as indicated by the straight dashed line. This dashed line represents the maximum discharge per unit width (q_e) that can be sustained at a particular value of the parameter $\left(\left[g\left(S_s - 1\right)\right]^{1/2} d_e^{3/8} h^{9/8}\right)$. The discharge indicated by the dashed line is termed the **equilibrium maximum discharge**. Any increase in discharge beyond the equilibrium value will result in an increase in water depth.

The scatter of measurements beneath the dashed line is pronounced, and this indicates that the discharge calculated for those measurements was less than could be tolerated by the depth at that location. Points just beneath the dashed line might be locations where the present bottom was eroded by discharges slightly greater than those measured during the field exercises. Many of the data points well below the line came from inlet cross sections either slightly seaward of the jetties where depths are controlled by waves and longshore currents or landward of the entrance channel where the tidal current is insufficient to scour the channel and depths have been increased by dredging.

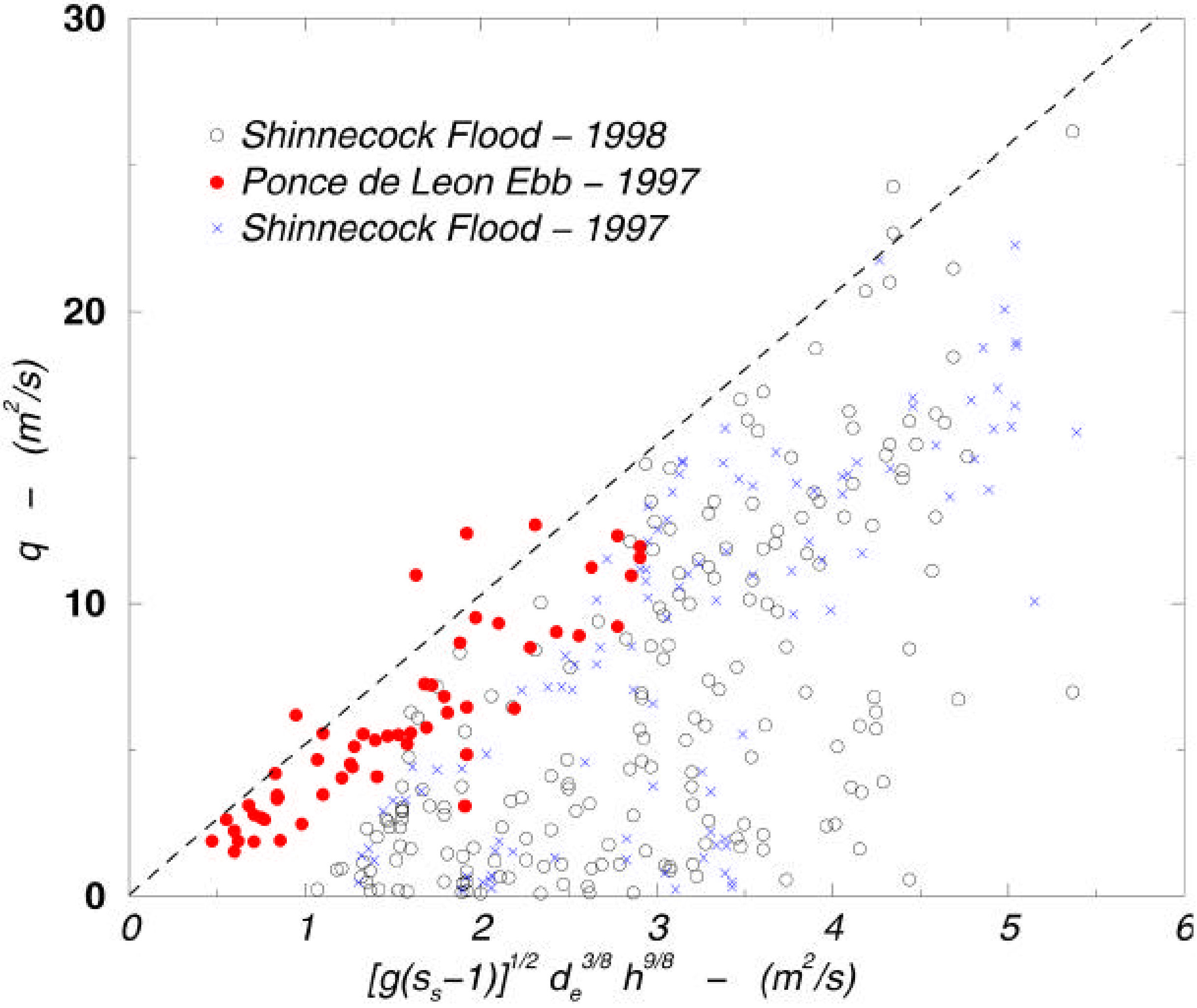

Figure 1. Field data from two dual-jettied inlets

Another explanation for data scatter below the dashed line is that depths at some of the locations are scoured by a different cross-channel flow distribution that occurs during the reverse maximum tidal flow. Finally, there is the possibility that some of the depths are the result of scouring that occurred during episodic events such as storm surges or river discharge combined with ebb flow. Regardless of the reason, depths associated with data points below the dashed line are not in equilibrium with the measured discharge. In other words, these depths would be able to accommodate increased flow discharge without additional scouring of the bottom.

The dashed line in Figure 1 corresponds to $C_e = 5.12$ in Equation 4, which can now be expressed as an empirical equation for equilibrium maximum discharge per unit width, i.e.,

$$q_e = 5.12\left[g\left(S_s - 1\right)\right]^{1/2} d_e^{3/8}\, h^{9/8} \tag{6}$$

For a given noncohesive sediment, there is an **equilibrium scour depth**, h_e, associated with the equilibrium discharge q_e. The depth h_e is taken relative to the tide level at maximum discharge. An expression for h_e is obtained by rearranging Equation 6 to get

$$h_e = \frac{0.234\, q_e^{8/9}}{\left[g\left(S_s - 1\right)\right]^{4/9}\, d_e^{1/3}} \tag{7}$$

Although it might be possible to have depths greater than the equilibrium scour depth, these depths would have to be caused by some process other than the maximum discharge at that location. Estimates of equilibrium scour depth from Equation 7 should be considered conservative because the estimates represent the outer envelope of the field data. In reality, the maximum discharge per unit width may not persist long enough to allow scoured depths to reach the predicted equilibrium depth.

Finally, substitution of the value of C_e into Equation 3 and rearranging provides a relationship for mean velocity at a location in terms of the equilibrium depth and sand parameters, i.e.,

$$\overline{V} = 5.12\left[g\left(S_s - 1\right)\right]^{1/2}\, d_e^{3/8}\, h_e^{1/8} \tag{8}$$

Plots of Equations 7 and 8 for a variety of quartz sand sizes are given in Figures 2 and 3. These plots show equilibrium depth (h_e) as a function of equilibrium discharge per unit width (q_e) and mean flow velocity ($\overline{V}$), respectively, for a range of quartz sand median grain-size diameters. The plots illustrate the effect of grain-size diameter on the equilibrium depth. As expected, channels with coarser sand have less depth at equilibrium under the same flow condition.

APPLICATION OF THE EQUILIBRIUM DISCHARGE DEPTH RELATIONSHIP: The semiempirical relationships for equilibrium depth as a function of sand parameters and discharge per unit width (Equation 7) or mean velocity (Equation 8) give depth estimates that are probably conservative, i.e., deeper than might actually occur for the specified discharge. Use of these formulas should be restricted to regions in the inlet throat where the scour appears to be caused by the maximum discharge. For example, depths in scour holes formed by vortices associated with flow separation will not be predicted by the equilibrium discharge depth relationship. In addition, the equations do not account for depth increases because of wave action in the channel.

Important Note: Correct use of the predictive equations in this CETN requires that all variables be given in a consistent set of units. In particular, sediment grain size needs to be expressed in the same length unit used for q_e and g in the equations (meters in the following examples).

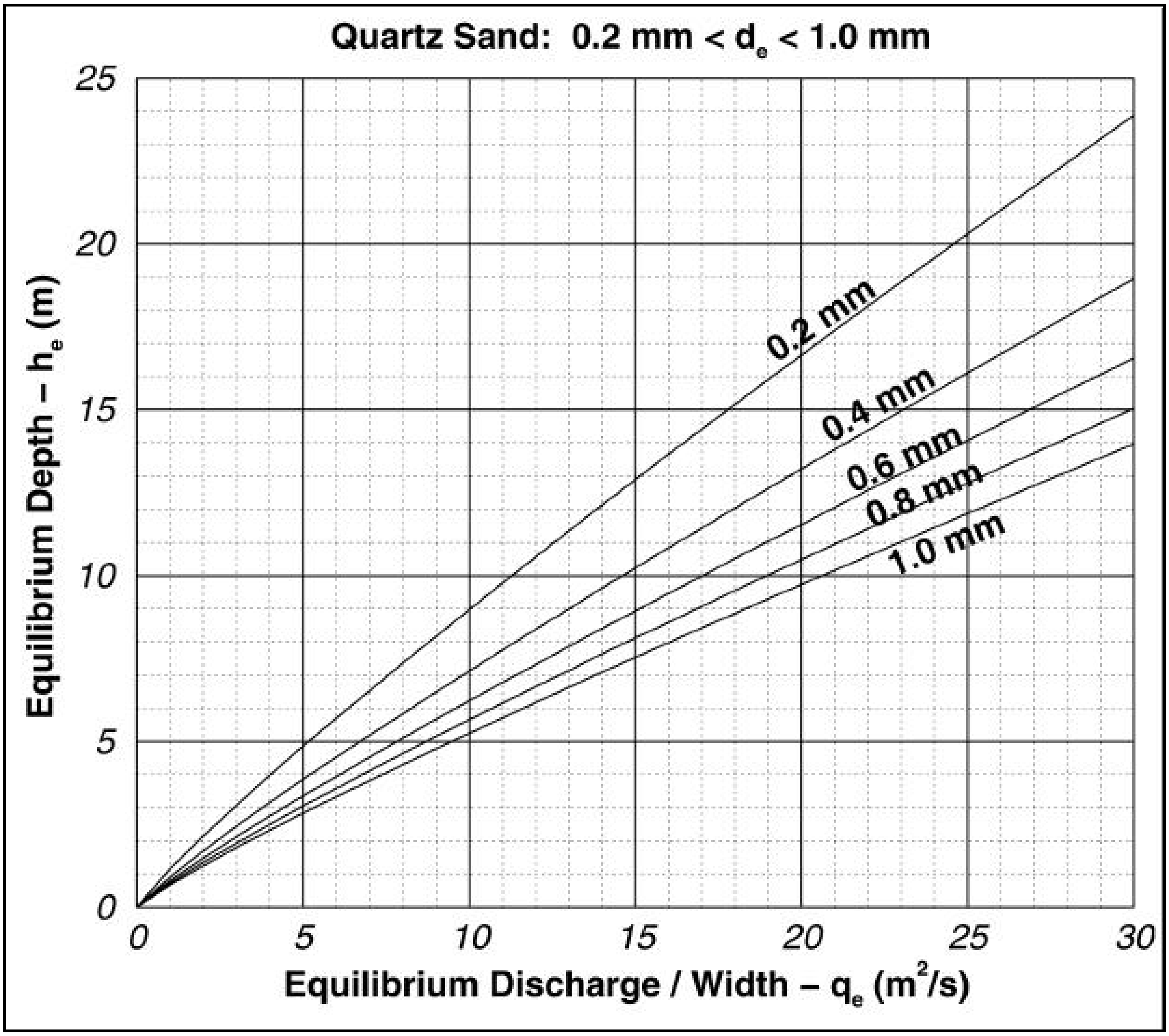

Figure 2. Equilibrium depth as a function of equilibrium discharge

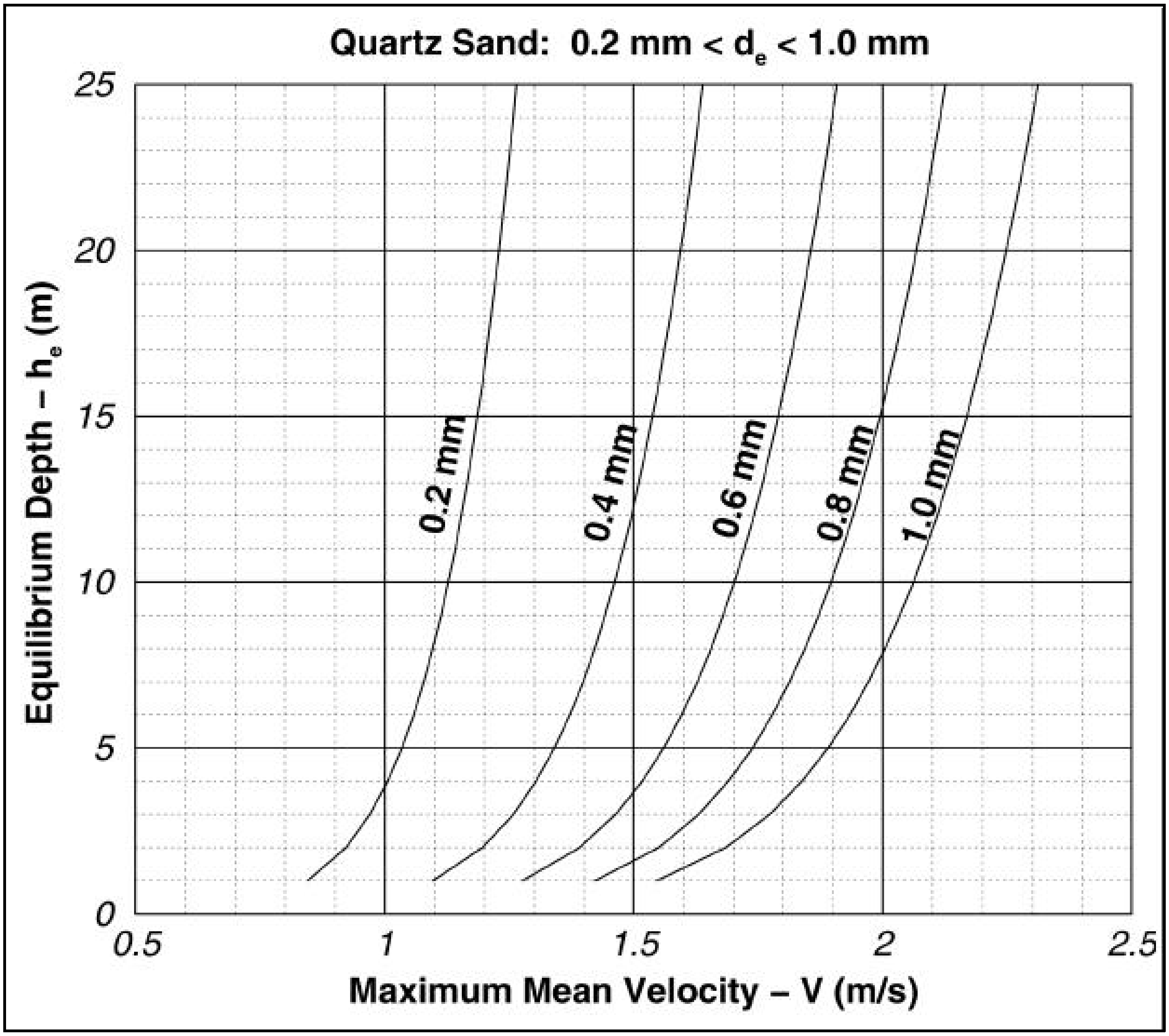

Figure 3. Equilibrium depth as a function of maximum mean velocity

Example 1: Scour at Ventura Harbor, California. During storm conditions, longshore currents flowing through a narrow gap between the North Jetty and Detached Breakwater at Ventura Harbor caused a scour hole with maximum depth of 9.5 m below mean lower low water (mllw). Figure 4 shows a plan view of the navigation structures. The scour hole was filled with quarrystone and protected with a stone sill having top elevation at -4.5 m mllw (Hughes and Schwichtenberg 1998). The sand in the vicinity is quartz with a median grain size near d_e = 0.2 mm (or d_e = 0.0002 m).

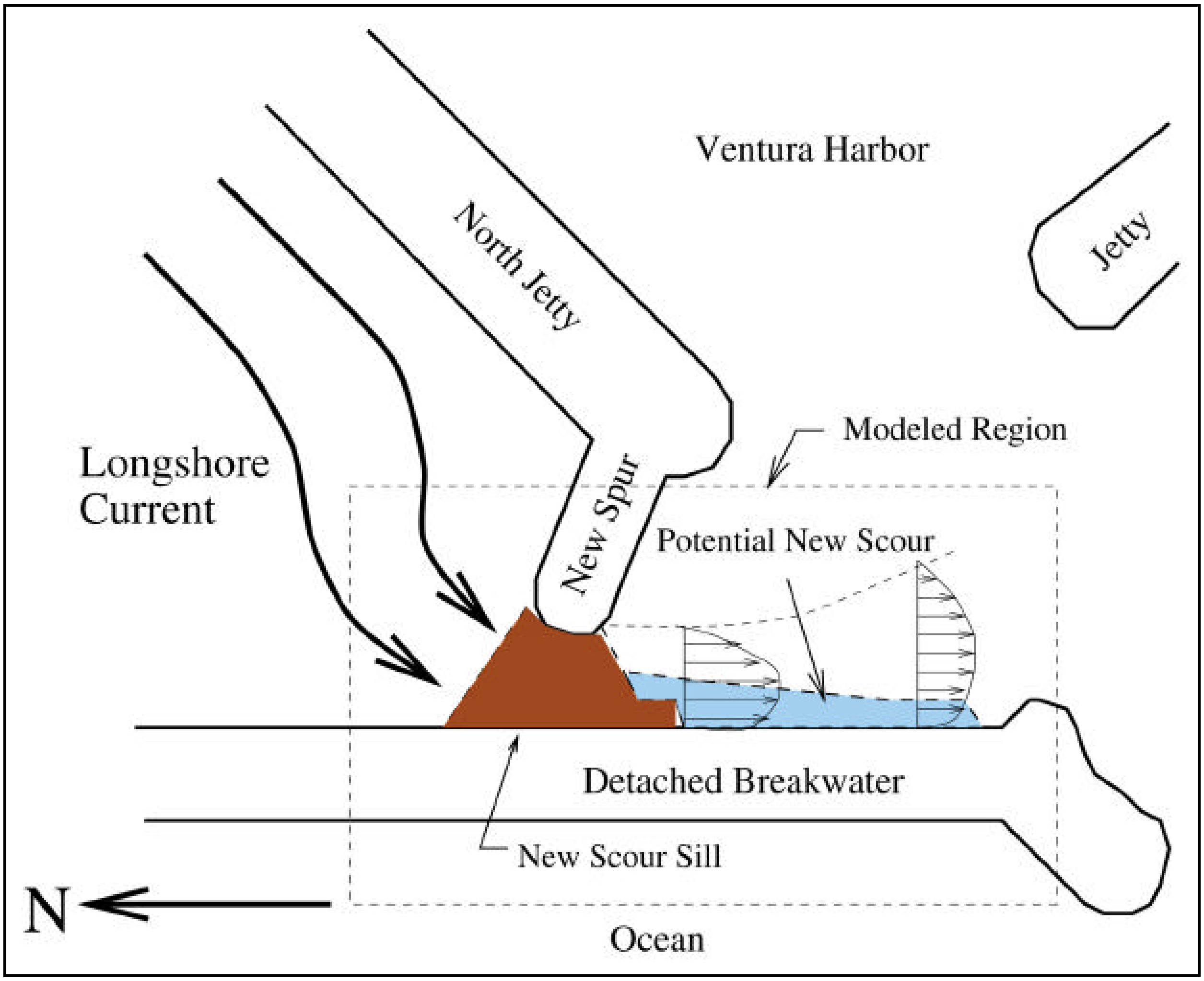

Figure 4. Ventura Harbor, California, navigation structures

An estimate of the equilibrium discharge per unit width corresponding to a scour depth of 9.5 m is determined from Equation 6 using a grain size of 0.2 mm, i.e.,

$$q_e = 5.12\left[\left(9.807\, m/s^2\right)(2.65-1)\right]^{1/2} (0.0002\, m)^{3/8} (9.5\, m)^{9/8} = \underline{10.63\, m^2/s}$$

Alternately, a value for q_e can be read directly from the plot in Figure 2 by finding the depth of 9.5 m on the vertical axis, extending a horizontal line to intersect with the "d_e = 0.2 mm" line, and reading the corresponding discharge on the horizontal axis. Figure 2 gives a value of

$$q_e \approx 10.5 m^2/s$$

The maximum mean velocity corresponding to the equilibirum discharge is found from Equation 5 as

$$\overline{V}_{scour} = \frac{q_e}{h} = \frac{10.5 m^2/s}{9.5 m} = \underline{1.1 m/s}$$

The same result could have been read directly from the curve in Figure 3 corresponding to d_e = 0.2 mm. Also note these estimates assume maximum discharge occurring at mllw.

Once the scour hole was filled in and capped at the -4.5-m mllw elevation, a similar storm producing the same discharge through the gap between the North Jetty and Detached Breakwater will produce an increased mean velocity given by

$$\overline{V}_{sill} = \frac{q_e}{h} = \frac{10.5 m^2/s}{4.5 m} = \underline{2.4 m/s}$$

Visual estimates of flow speed through the gap during a storm after the sill was placed were on the order of 2.5-3.0 m/s.[1] The estimate of increased mean velocity is useful for determining the absolute minimum scour blanket stone size if scour holes are filled and covered over with stone.

Example 2: Freshwater Discharge. A recent modification to the jetty system of a fictitious tidal inlet on the Pacific Coast resulted in ebb-flow redirection and the formation of a 6-m-deep scour hole adjacent to one of the jetties. The bed material is quartz sand with d_e . 0.6 mm (0.0006 m). The scour hole in its present configuration does not threaten the jetty toe.

During normal conditions, only minor freshwater runoff empties into the bay and flows out the entrance channel. However, during El Niño years, a large quantity of freshwater runoff flows into the bay via flood channels. It is estimated that the maximum freshwater runoff will increase the *discharge per unit width* through the inlet throat by 3 m^2/s for a period lasting several days.

> What will be the maximum depth of the scour hole as a result of the freshwater surcharge?

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[1] Personal Communication, 1998, B. R. Schwichtenberg, U.S. Army Engineer District, Los Angeles, Los Angeles, CA.

Using the plot in Figure 2, an equilibrium scour depth (relative to tide elevation of maximum discharge) of h_e = 6 *m* on the vertical axis corresponds to a discharge per unit width of q_e . 9.7 m^2/s for sand with median diameter of 0.6 mm = 0.0006 m. Adding the freshwater discharge of 3 m^2/s gives a new discharge of 12.7 m^2/s. From the same plot, this increased discharge corresponds to a new equilibrium scour depth of 7.7 m, or a 1.7-m depth increase.

This estimate is likely conservative because the combined freshwater and ebb-tidal flow are maximum for a relatively short time during each tidal cycle. Whether or not several days will be sufficient time to reach a new scour equilibrium is unknown. Also note that any effect of flow stratification because of the influx of less dense fresh water is not considered.

Example 3: Nonquartz Sediment. The curves in Figures 2 and 3 pertain to quartz sand with specific gravity of 2.65. Estimates for inlets having nonquartz sediments must use Equation 7 or 8.

For example, consider another fictitious tidal inlet at a location where the sediment is primarily broken shell material having specific gravity of $S_s = \tilde{n}_s/\tilde{n}_w$ = 2.4 and d_e = 0.5 *mm* = 0.0005 *m*.

> What will be the equilibrium depth corresponding to a discharge per unit width of 8 m^2/s?

Substituting numerical values for the variables in Equation 7 yields

$$h_e = \frac{0.234\left(8\,m^2/s\right)^{8/9}}{\left[\left(9.807\,m/s^2\right)(2.4-1)\right]^{4/9}\,\left(0.005\,m\right)^{1/3}} = \frac{0.234\,(6.35)}{(3.204)(0.0794)}\,m = \underline{5.8\,m}$$

An equivalent estimate for quartz sand produced an equilibrium scour depth about 0.4 m less than found for the shell sediment. This difference is probably less than the error that might be expected for this primitive estimation technique, so use of Figures 2 and 3 for nonquartz sediments should not introduce too much error except for exotic sediments that are either extremely heavy or nearly neutrally buoyant.

ADDITIONAL INFORMATION. Questions about this CETN can be addressed to Dr. Steven A. Hughes (Voice: 601-634-2026, FAX: 601-634-3433, e-mail: *s.hughes@cerc.wes.army.mil*). For information about the Coastal Inlets Research Program, please contact the Program Manager, Mr. E. Clark McNair (Voice: 601-634-2070, e-mail: *mcnairc@ex1.wes.army.mil*). Dr. Nicholas Kraus provided a beneficial review of this CETN. This technical note should be cited as follows:

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